General model of depolarization and transfer of polarization of singly ionized atoms by collisions with hydrogen atoms

M. Derouich$^{1,2}$

$^1$Astronomy Department, Faculty of Science, King Abdulaziz University, 21589 Jeddah, Saudi Arabia
$^2$Sousse University, ESSTHS, Lamine Abbassi street, 4011 H. Sousse, Tunisia

Abstract
Simulations of the generation of the atomic polarization is necessary for interpreting the second solar spectrum. For this purpose, it is important to rigorously determine the effects of the isotropic collisions with neutral hydrogen on the atomic polarization of the neutral atoms, ionized atoms and molecules. Our aim is to treat in generality the problem of depolarizing isotropic collisions between singly ionized atoms and neutral hydrogen in its ground state. Using our numerical code, we computed the collisional depolarization rates of the $p$-levels of ions for large number of values of the effective principal quantum number $n^*$ and the Unsöld energy $E_p$. Then, genetic programming has been utilized to fit the available depolarization rates. As a result, strongly non-linear relationships between the collisional depolarization rates, $n^*$ and $E_p$, are obtained, and are shown to reproduce the original data with accuracy clearly better than 10%. These relationships allow quick calculations of the depolarizing collisional rates of any simple ion which is very useful for the solar physics community. In addition, the depolarization rates associated to the complex ions and to the hyperfine levels can be easily derived from our results. In this work we have shown that by using powerful numerical approach and our collisional method, general model giving the depolarization of the ions can be obtained to be exploited for solar applications.

Keywords: Scattering – Sun: photosphere – atomic processes – line: formation – line: profiles – polarization

1. Introduction
The transitions between levels of solar ions caused by anisotropic scattering of the incident radiation induce population imbalances and quantum coherences.

*M. Derouich
Email address: derouichmoncef@gmail.com (M. Derouich$^{1,2}$)
among the Zeeman sublevels. Population imbalances and quantum coherences are usually called atomic polarization. The second solar spectrum (SSS), which is the spectrum of the linear polarization observed near the limb of the Sun, is the observational signature of the atomic polarization. The term SSS was coined by Ivanov (1991). It is also usually referred to as spectrum of the scattering polarization since it is due to coherent scattering processes (e.g. Trujillo Bueno 2001).

The SSS is modified by solar magnetic fields via the Hanle effect. Such a modification allows diagnostics of the magnetic fields. In fact, Hanle effect diagnostics of hidden, mixed-polarity magnetic fields at sub-telescopic scales require confronting the discrepancy between the polarization calculated in the absence of magnetic fields and the observed linear scattering polarization (e.g. Stenflo 1982; Landi Degl’Innocenti 1983; Sahal-Bréchot et al. 1986; Stenflo 2004; Trujillo Bueno et al. 2004; Derouich et al. 2006; Faurobert et al. 2009). Reliable diagnostics consists in solving numerically the coupled set of equations of the radiative transfer and the statistical equilibrium of a multilevel atomic system taking into account radiative and collisional processes. The collisional rates, which are often poorly known, are a fundamental ingredient for realistic diagnostics. Sometimes the information encoded in accurate solar spectropolarimetric observations would be inaccessible if the effect of the collisions is unknown (e.g. Derouich et al. 2006; Derouich et al. 2007). There is a need for theory and numerical modeling of collisional depolarization of spectral lines by nearby hydrogen atoms. We notice that the collisions with hydrogen are dominant because it is the most abundant atom in the solar photosphere where the SSS is formed.

In this context, we aim to generalize our numerical modeling of Derouich et al. (2004) to provide collisional data for any ionic level. The paper is organized as follows. Section 2 gives a brief review of the theoretical background. In Section 3, we formulate the problem. Section 4 explains the basic definitions and notations, describes the collisional data employed in the context of this work, and provides the results for simple atoms without hyperfine structure. Simple atoms with hyperfine structure and complex atoms are presented in sections 5 and 6. Finally, the conclusion of the paper is presented in section 7.

2. Brief theoretical background

In real plasmas like the solar atmosphere, emitting ions suffer the effects of collisions with nearby abundant particles. To correctly extract informations contained in the solar observations would be inaccessible, the effect of the collisions must be taken into account. During the 2000s, Derouich, Sahal-Bréchot and Barklem (DSB) developed a semi-classical theory allowing precise and quick calculations of the depolarization and polarization transfer rates by collisions with neutral hydrogen (see for example Derouich et al. (2003a,b, 2005a,b); Derouich 2004; Derouich and Barklem (2007)). The DSB approach is based on the Anstee-Barklem-O’Mara (ABO) theory concerned with collisional line broaden-

An important result which justifies the use of the DSB semi-classical approach is that the collisional depolarization and polarization transfer probabilities depend exclusively on the intermediate range of the interatomic separations. We have shown that a modification of the interaction potential values by a Gaussian magnification factor, for the close or long-range regions of the interaction, does not practically change the values of the calculated collisional probabilities (Derouich et al. 2003a, 2004). In fact, we have found that a significant effect on the collisional depolarization and polarization transfer probabilities takes place only in the intermediate range of the interatomic separations. The sophisticated quantum chemistry approach is different to our semi-classical approach only in the short-range regions of the interactions. Since close interaction regions do not influence the values of the depolarization rates, we obtain good results compared to the available quantum chemistry ones (~10% of accuracy).

In addition, the semi-classical approach is very useful for heavy and/or complex atoms/ions like Fe I, Ti I, NdII, EuII, CeII, ZrII, etc., whose collisional rates cannot be presently obtained via quantum chemistry methods. The spectral lines of such atoms/ions show significant polarization peaks in many spectral lines (see the atlases by Gandorfer 2000, 2002, 2005).

Although the fact that this theory is of semi-classical nature, the close coupling is taken into account and the time-dependent Schrödinger equations is solved.

3. Formulation of the problem

For neutral atoms, the DSB and ABO theories give general results for neutral atoms however they lose their generality in the case of ionized atoms. This is because, for all neutral atoms, the Unsöld energy $E_p$ can be replaced by a constant value $E_p = -4/9$ atomic units (see, e.g., Anstee 1992, Barklem & O’Mara 1998; Derouich et al. 2003a).

On the contrary, while ABO and DSB theories can be applied for any singly ionized atom, the calculation of the parameter $E_p$ is required for each level which implies that the results are specific for each level of a particular ion. The interaction energy associated to the interaction of a singly ionized atom with a hydrogen atom in its ground state is given by Equation (4) of Derouich et al. (2004). After some derivations and by using the Unsöld approximation, the expression of the interaction potential becomes dependent on the parameter $E_p$, as shown by Equation (9) of Derouich et al. (2004). The variation of the depolarization rates as a function of $E_p$ is given by Derouich et al. (2004) who concluded that one has to determine $E_p$ before going to the calculation of the depolarization rates. This limitation substantially restrict the generalization of the results of Derouich et al. (2004). In fact, two steps are needed in the calculation of the depolarization rates for the levels of ions.
1. Step 1: one must determine $E_p$ directly for each state of each ion via the expression:

$$E_p = -\frac{2 < p^2 >}{C_6},$$  \hspace{1cm} (1)

where $< p^2 >$ is the mean square distance between the optical electron and the perturbed ion core,

$$< p^2 > = \frac{n^*}{Z^2} [5n^* + 1 - 3l(l + 1)].$$  \hspace{1cm} (2)

$l=1$ is the orbital momentum for $p$-states, $Z=2$ for singly ionized atoms and $n^*$ is the effective quantum number (see Derouich et al. 2004). $C_6$ is the Van der Waals coefficient given by the standard relationship (see for instance Goodisman 1973 and Derouich et al. 2004).

2. Step 2: Then, the value of $E_p$ is included in the expression of the interaction energy. After that, the probabilities of collisions are obtained by solving the semi-classical differential coupled equations which are derived from the time-dependent Schrödinger equation (Derouich et al. 2003a). To obtain the depolarization rates, one thus must perform the integration of the probabilities of collisions over the impact-parameter $b$ and over a Maxwell distribution of velocities $f(v)$ for a temperature $T$ of the medium.

In practice, solar physicists can calculate the value of $E_p$ (step 1) but it might be quite complicated for them to determine the depolarization rates by using the collision theory (step 2). The main goal of this work is to overcome numerically the difficulty pointed out in the step 2 to give the possibility of the determination of the depolarization rates by completing only step 1.

4. Definitions, notations and numerical results

4.1. Definitions and notations

We denote the atomic levels by $(\alpha J)$ where $J$ is the total angular momentum of the level and $\alpha$ represents the other quantum numbers necessary to define the electronic level of the ion. The atomic states $(\alpha J)$ are described by the tensorial components $\rho^k_q(\alpha J)$ of the atomic density matrix. The number $k$ is the tensorial order where $0 \leq k \leq 2J$ and $-k \leq q \leq k$ quantifies the coherences between the levels and the sublevels (e.g. Fano 1963; Omont 1977; Sahal-Bréchet 1977; Landi Degl’Innocenti & Landolfi 2004).

In the case of interactions of singly ionized ions with neutral hydrogen, the inelastic and super-elastic collisions between two different electronic levels are negligible. The indice $\alpha$ is omitted from now on for the sake of brevity. We apply the semi-classical theory developed by Derouich et al. (2004) and the associated numerical code to obtain, the all non zero collisional rates of any $p$-level.
At the solar photosphere where the SSS is formed, the dominant collisions with neutral hydrogen are isotropic. Due to this isotropy, the depolarization and polarization transfer rates are $q$-independent (e.g. Sahal-Bréchot 1977; Derouich et al. 2003a, Landi Degl’Innocenti & Landolfi 2004). The rates are obtained in the tensorial basis. We denote by $D_k(J)$ the depolarization rates due to purely elastic collisions; the expression of $D_k(J)$ is given for example by equations (7) and (9) of Derouich et al. (2003a). $C_k(J \rightarrow J')$ are the polarization transfer rates due to collisions between the initial electronic state ($J$) and the final state ($J'$) respectively (see Equation 3 of Derouich et al. 2003b; Landi Degl’Innocenti & Landolfi 2004).

According to Equation (1) of Derouich et al. (2004) (see also Equation (1) of Sahal-Bréchot et al. (2007) and Sahal-Bréchot (1977)), the variation of $\rho_k^q(J)$ due to isotropic collisions is:

$$\frac{d}{dt} \rho_k^q(J)_{\text{coll}} = -\left[ \sum_{J' \neq J} \zeta(J \rightarrow J') + D_k(J) \right] \times \rho_k^q(J) + \sum_{J' \neq J} C_k(J' \rightarrow J) \times \rho_k^q(J')$$

where $\zeta(J \rightarrow J')$ are the fine structure transfer rates given by (Equation 5 of Derouich et al. 2003b):

$$\zeta(J \rightarrow J') = \sqrt{\frac{2J' + 1}{2J + 1}} \times C_0^a(J \rightarrow J').$$

We notice that this rate equation is slightly different from the rate equation (7.101) of Landi Degl’Innocenti & Landolfi (2004) (page 343). To avoid confusion, we will demonstrate that these equations are not in contradiction, they are similar.

In fact, as it was mentioned for example in Derouich (2008) (see also Derouich & Ben Abdallah (2009)), there is a multiplicity factor equal to $\sqrt{\frac{2J' + 1}{2J + 1}}$ between the collisional rates $C_k(J \rightarrow J')$ calculated by our method (and defined for instance in Sahal-Bréchot (1977); Derouich et al. (2004); Sahal-Bréchot et al. (2007)) and the collisional rates $[C_k(J \rightarrow J')]_{LL}$ defined by Landi Degl’Innocenti & Landolfi (2004). One has:

$$C_k(J' \rightarrow J) = \sqrt{\frac{2J' + 1}{2J + 1}} [C_k(J' \rightarrow J)]_{LL}$$

$$\sqrt{\frac{2J' + 1}{2J + 1}} C_0^a(J \rightarrow J') = [C_0^a(J \rightarrow J')]_{LL}$$

Since for the depolarization rate one has $J = J'$ and thus $\sqrt{\frac{2J' + 1}{2J + 1}} = 1$, the $D_k(J)$ is the same in both definitions. According to Landi Degl’Innocenti &
Landolfi (2004), the rate equation is:

\[
\left[ \frac{d \rho^k(J)}{dt} \right]_{\text{coll}} = - \left[ \sum_{J' \neq J} [C^0(J \rightarrow J')]_{\text{LL}} + D^k(J) \right] \times \rho^k(J) (7)
\]

\[
+ \sum_{J' \neq J} \sqrt{\frac{2J' + 1}{2J + 1}} [C^k(J' \rightarrow J)]_{\text{LL}} \times \rho^k(J')
\]

It is easy to see that, using our collisional rates, the rate equation of Landi Degl’Innocenti & Landolfi (2004) (Equation 7 of this paper) becomes exactly the same as the rate equation given in the present paper (Equation 3).

In our present paper, to ensure continuity, we adopt the definitions related to Equation 3 since they are known from the older work of Sahal-Bréchot (1977) and they follow the definitions adopted in our previous papers. In any case, this is nothing more than a difference in definitions. Nevertheless, this difference should be taken into account when writing the variation of the density matrix in order to calculate the polarization signals correctly. For this reason, Equation 3 is necessary for a reader who wants to exploit our work paper.

In our calculations, we consider a simple ion with one optical electron in the external shell with an orbital angular momentum \( \vec{l} \) and spin \( \vec{s} \) (s=1/2). The cases of complex atoms and atoms with hyperfine structure can be easily derived as it is explained in the last sections of this paper.

Our results are concentrated on the \( p \)-states (\( l=1 \)) case which is frequently encountered in the SSS. In the LS coupling scheme, the total angular momentum \( \vec{J}=\vec{l}+\vec{s} \) and one obtains \( J=1/2 \) or \( J=3/2 \). These \( J \)-values are associated to the states \( ^2P_{J=\frac{1}{2}} \) and \( ^2P_{J=\frac{3}{2}} \).

We notice that all the collisional rates are given in \( s^{-1} \). Since the impact approximation is well satisfied, the rates are proportional to \( n_H \) which is the neutral hydrogen density in \( \text{cm}^{-3} \). They depend on the temperature \( T \) given in Kelvins. The collisional rates are usually expressed as (e.g. Derouich et al. 2003a; Derouich et al. 2004)

\[
D^k(J,T) = D^k(J,T=5000K) \times \left( \frac{T}{5000} \right)^{1+\lambda} (8)
\]

\[
C^k(J \rightarrow J', T) = C^k(J \rightarrow J', T=5000K) \times \left( \frac{T}{5000} \right)^{1+\lambda} (9)
\]

where \( T = 5000K \) is the reference temperature for which collisional rates are tabulated and \( \lambda \) is the so-called velocity exponent. DSB and ABO found that \( \lambda \) has a limited range of variation around 0.25 and, as a result, that collisional rates have typical temperature dependence of \( T^{0.38} \).

4.2. Numerical results

Grids of collisional rates are computed at \( T = 5000K \) for the effective principal quantum number \( n^* \) of the \( p \)-states in the interval [1.5, 3]. In addition, we calculated for each \( n^* \), grids of collisional rates for Unsöld energy \( E_p \) ranged...
in the interval [-1.2, -0.1] (in atomic units). We adopt a step size of 0.1 when varying \( n^* \) and \( E_p \). As a result of our calculations, we obtain three dimensional tables giving the collisional rates with \( n^* \) and \( E_p \).

Using these tables, we applied our Genetic Programming (GP)-based method in order to infer analytical relationships between collisional rates, \( n^* \) and \( E_p \). This method has been used in Derouich et al. (2015). We minimize, in a least squares sense, the difference between the tabulated rates and rates obtained from the GP relationships. The predicted relationships from the GP-based model are compared with the available data of the \( p \)-state of the Ca II in order to estimate the accuracy of our results.

The collisional depolarization/transfer of polarization rates are in the left-hand side of the relationships. The second term in the right hand side of the relationships is a function of \( n^* \) and \( E_p \). For \( J = \frac{1}{2} \), only the value \( k = 1 \) is possible. However for \( J = \frac{3}{2} \), \( 0 \leq k \leq 3 \). In addition, by definition, \( D^{k=0}(J) = 0 \). Thus the non-zero depolarizations rates associated to the levels \( 2P_J = \frac{1}{2} \) and \( 2P_J = \frac{3}{2} \) are:

\[
D^1\left(\frac{1}{2}\right)(T = 5000K)/(n_H \times 10^{-9}) = X - \frac{Y^2}{49} + \frac{(X - 1.)}{(28Y - Y \times X)} - \frac{Y}{(2. - \frac{Y}{(X \times Y - 2X)})} \tag{10}
\]

where \( X = n^*_p \) and \( Y = -E_p > 0 \). For instance, let us consider the case of the \( p \)-state of the Ca II where \( n^*_p = 2.49 \) and \( E_p = -0.544 \). According to the Equation 12 of Derouich et al. (2004), \( D^1\left(\frac{1}{2}\right)(T = 5000K)/(n_H \times 10^{-9}) = 2.48 \text{ s}^{-1} \). Equation (10) gives \( D^1\left(\frac{1}{2}\right)(T = 5000K)/(n_H \times 10^{-9}) = 2.34 \text{ s}^{-1} \). Thus, the relative error is less than 6% which demonstrates how good is the precision of our GP method of fitting.

\[
D^1\left(\frac{3}{2}\right)(T = 5000K)/(n_H \times 10^{-9}) = \left[ 5. + \frac{X}{(10Y + \frac{5}{Y}X \times X)} \right] \times \left[ \frac{7.}{(3X + X^2)} + \frac{7.}{(10 + 2X)} \right] \times \left[ \frac{1.}{(Y + 5. - \frac{5}{Y})} \right] \tag{11}
\]

In the case of the \( p \)-state of the Ca II, the relative error on the determination of \( D^1\left(\frac{3}{2}\right) \) is less than 4% with respect to the reference value given in Equation 13 of Derouich et al. (2004).

\[
D^2\left(\frac{3}{2}\right)(T = 5000K)/(n_H \times 10^{-9}) = \frac{X^2}{Y - 3X + \frac{5}{Y}} + X^2 - Y \times X + Y \times \frac{Y}{(14. + 7. \times Y)} \tag{12}
\]
With respect to the reference value given in Equation 13 of Derouich et al. (2004), the relative error on the determination of \( D^{3/2} \) is \( \simeq 2 \% \) in the case of the \( p \)-state of the Ca II.

\[
D^{3/2}(T = 5000 \text{K})/(n_H \times 10^{-9}) = \frac{(X^2 + \frac{8}{5})}{(7 - \frac{3}{5} - X + Y)} \times (1 + X - \frac{Y}{(Y + 2.3 - 0.5)} \quad (13)
\]

By comparing to the reference value of \( D^{3/2}(T = 5000 \text{K}) \) given in Equation 13 of Derouich et al. (2004), the relative error is \( \simeq 2 \% \) in the case of the \( p \)-state of the Ca II.

Concerning the non-zero polarization transfer rates between the levels \( 2P_{J=1/2} \) and \( 2P_{J=3/2} \), only the rates \( C^0(1/2 \rightarrow 3/2), C^0(3/2 \rightarrow 1/2), C^1(1/2 \rightarrow 3/2), \) and \( C^1(3/2 \rightarrow 1/2) \) are non-zero,

\[
C^0(1/2 \rightarrow 3/2)(T = 5000 \text{K})/(n_H \times 10^{-9}) = \frac{X^2}{(6 + \frac{8}{5} - \frac{X}{5} + Y)} \times (X + \frac{(5 - Y)}{(5 + Y)}) \quad (14)
\]

By comparing to the reference value of \( C^0(1/2 \rightarrow 3/2)(T = 5000 \text{K}) \) given in Equation 17 of Derouich et al. (2004), we found a relative error \( \simeq 8 \% \) for the case of the \( p \)-state of the Ca II. Note that in Derouich et al. (2004), the polarization transfer rates are denoted by \( D^k(J' \rightarrow J, T) \) instead of the notation \( C^k(J' \rightarrow J, T) \) adopted here.

\[
C^1(1/2 \rightarrow 3/2)(T = 5000 \text{K})/(n_H \times 10^{-9}) = \frac{(X+1)\times(X+Y)}{(X-1)\times(X+Y)} + \frac{(2+Y)}{(5-X)} + 6 \times (X^2 - Y \times X - 23/2. \quad (15)
\]

By comparing to the reference value of \( C^1(1/2 \rightarrow 3/2)(T = 5000 \text{K}) \) given in Equation 17 of Derouich et al. (2004), we found a relative error is less than 3 %.

Only the excitation collisional transfer rates \( C^0(1/2 \rightarrow 3/2) \) and \( C^1(1/2 \rightarrow 3/2) \) are given. However, it is straightforward to retrieve the values of the deexcitation collisional rates \( C^0(3/2 \rightarrow 1/2) \) and \( C^1(3/2 \rightarrow 1/2) \) by applying the detailed balance relation:

\[
C^k(J_u \rightarrow J_l, T) = 2J_l + 1 \exp\left(\frac{E_{J_u} - E_{J_l}}{k_B T}\right) C^k(J_l \rightarrow J_u, T) \quad (16)
\]
where $J_l=1/2$ (lower level) and $J_u=3/2$ (upper level); $E_J$ being the energy of the level ($J$) and $k_B$ the Boltzmann constant.

Thanks to the relationships given here, any collisional rates can be obtained by simply determining the value of $n^*$ and $E_p$, i.e. by performing the step 1 as explained in the previous section without the need to step 2. This is the main result of this work.

5. Hyperfine structure

In the typical solar conditions where the temperature is about 5700K, the inverse of the typical time duration of a collision between a hydrogen atom and the perturbed ion is $1/\tau \sim 10^{13}$ s$^{-1}$. In these conditions, the hyperfine splitting is usually much smaller than $1/\tau$ and therefore one can assume that the nuclear spin is conserved during the collision. This is the frozen nuclear spin approximation.

In the framework of the frozen nuclear spin approximation, the depolarization and polarization transfer rates of hyperfine levels are given as a linear combination of the rates $D^k(J)$ and $C^k(J \rightarrow J')$ associated to the levels of the fine structure (e.g. Nienhuis 1976 and Omont 1977). Using the results of this work and after calculating simple coefficients of the linear combination which are illustrated in Derouich et al. (2005b) and given by Equations (4.6) of Nienhuis (1976) and (4.48) of Omont (1977), it is possible to perform sufficiently accurate computation of the needed collisional depolarizing and polarization transfer rates for all levels of ions with hyperfine structure.

6. Complex atoms

Our relationships given by Equations 7–12 can be directly applied to obtain depolarization and polarization transfer rates of the lines of singly ionized ions like Be II, Mg II, Ca II, Sr II, and Ba II in their $p$-states since they are simple ions because they have only one valence electron above a filled subshell. In contrast, the electronic configuration of a complex ion has one valence electron above an incomplete (open) subshell within the core. Derouich et al. (2005a) provided, from a conceptual and numerical point views, the physical model allowing, for the first time, the determination of the depolarization and polarization transfer rates of complex ions and atoms by using the rates of simple atoms. More details are also given in Derouich et al. (2005b), Derouich & Barklem (2007) and Sahal-Bréchet et al. (2007). In these works, we demonstrated that the depolarization and polarization transfer rates of complex atoms can be written as a linear combination of the rates of simple atoms. Therefore, it is straightforward to use our Equations 7–12 to infer rates of complex atoms by simply calculating  

\footnote{It is important however to not confuse this condition with the fact that the statistical equilibrium must be solved for the hyperfine levels when the inverse of the lifetime of the level is smaller than the hyperfine splitting, i.e. the hyperfine levels are separated.}
some algebra coefficients whose expressions are given in Derouich et al. (2005b), Derouich & Barklem (2007) and Sahal-Bréchot et al. (2007).

7. Conclusion

Derouich et al. (2004) proposed a semi-classical theory allowing the determination of the depolarization and polarization transfer rates of any simple ion, but one must apply the theory and the collisional numerical code in order to calculate these rates for each level of each ion. This makes the use of the results of Derouich et al. (2004) by the solar physics community rather limited. The purpose of the present paper is to generalize the semi-classical theory of Derouich et al. (2004) by giving general relationships allowing the determination of the depolarization and polarization transfer rates of any simple ion without the need to perform collisional calculations for each level. In addition, these relationships can be used to determine the depolarization/polarization transfer rates of the levels of hyperfine structured ions. Furthermore, these relationships allow sufficiently accurate determination of the rates associated to complex ions whose collisional rates cannot be presently obtained via standard quantum chemistry methods. We notice that the spectral lines of such ions show significant polarization peaks in many spectral lines of the SSS (see the atlases by Gandorfer (2000, 2002, 2005)).

All the results of this paper are performed thanks to powerful GP techniques. Genetic algorithms have been found to be very good at determining the global minimum in a space with many local minima. The details of the GP approach can be found in Derouich et al. (2015) and references therein.

Acknowledgements

I thank Dr. P. Barklem for stimulating scientific discussions and ongoing collaboration. I would like to thank referee 1 for the detailed constructive comments.

References

Derouich, M., 2004, Ph.D. Thesis, Paris VII-Denis Diderot University, France
Omont A., 1977, Prog. Quantum Electronics, 5, 69