Limits on Brane-World and Particle Dark Radiation from Big Bang Nucleosynthesis and the CMB

N. Sasankan
Center for Astrophysics, Department of Physics, University of Notre Dame
Notre Dame, IN 46556, USA nsasanka@nd.edu

Mayukh. R. Gangopadhyay
Center for Astrophysics, Department of Physics, University of Notre Dame
Notre Dame, IN 46556, USA
Theory Division, Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Kolkata- 700064, India

G. J. Mathews
Center for Astrophysics, Department of Physics, University of Notre Dame
Notre Dame, IN 46556, USA

M. Kusakabe
Center for Astrophysics, Department of Physics, University of Notre Dame
Notre Dame, IN 46556, USA

The term dark radiation is used both to describe a noninteracting neutrino species and as a correction to the Friedmann Equation in the simplest five-dimensional RS-II brane-world cosmology. In this paper we consider the constraints on both meanings of dark radiation based upon the newest results for light-element nuclear reaction rates, observed light-element abundances and the power spectrum of the Cosmic Microwave Background (CMB). Adding dark radiation during big bang nucleosynthesis (BBN) alters the Friedmann expansion rate causing the nuclear reactions to freeze out at a different temperature. This changes the final light element abundances at the end of BBN. Its influence on the CMB is to change the effective expansion rate at the surface of last scattering. We find that the BBN constraint reduces the allowed range for both types of dark radiation at 10 Mev to between $-12.1\%$ and $+6.2\%$ of the total background energy density at 10 Mev. Combining this result with fits to the CMB power spectrum, produces different results for particle vs. brane-world dark radiation. In the brane-world, the range decreases to $-6.0\%$ to $+6.2\%$. Thus, we find, that the ratio of dark radiation to the background total relativistic mass energy density $\rho_{\text{DR}}/\rho$ is consistent with zero although there remains a very slight preference for a positive (rather than negative) contribution.

Keywords: Brane-world; BBN; CMB; Dark radiation.

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1. Introduction

The term dark radiation is used both to describe a noninteracting neutrino species and as a correction to the Friedmann Equation in the simplest five-dimensional RS-II brane-world cosmology.\(^1\) In this paper we consider the constraints on both concepts of dark radiation. In a recent work\(^2\) we have considered detailed constraints on brane-world dark radiation based upon the newest results for light-element nuclear reaction rates, observed light-element abundances and the power spectrum of the Cosmic Microwave Background (CMB). In this paper we summarize those results and contrast the physics of brane-world vs. relativistic-particle dark radiation.

By particle dark radiation one usually means a relativistic neutrino-like particle that does not interact electromagnetically and therefore after weak decoupling mainly affects the background radiation density. On the other hand, brane-world dark radiation is a proposed solutions to the hierarchy problem among the fundamental forces. The introduction of a non-compact large extra dimension\(^1\) eliminates the hierarchy problem between the weak forces and the size of the compact extra dimensions. In the Randall and Sundrum model, the observed universe is a four-dimensional spacetime embedded in a five-dimensional anti-de-sitter space (AdS5).

The effect of particle dark radiation during the epoch of BBN and CMB has been studied in the context of constraining the effective number of neutrino species \(N_{\text{eff}}\)\(^3\)\(^\text{\textsuperscript{-\text{12}}}\). Positive or negative particle dark radiation can be associated with the uncertainty in the number of neutrino species \(\Delta N_\nu\). The standard model suggests that we have 3 types of neutrinos. An addition of dark radiation \((\rho_{\text{DR}})\) can be related to a corresponding value in \(\Delta N_\nu\) given by.

\[
\left( \sum_{i=e,\mu,\tau} \rho_{\nu_i} \right) + \rho_{\text{DR}} \equiv (3 + \Delta N_\nu) \rho_{\nu_e} \equiv N_{\text{eff}} \rho_{\nu_e} ,
\]

where \(\rho_{\nu_e}\) corresponds to the sum over neutrino plus anti-neutrino energy densities

\[
\rho_{\nu_e} = \frac{7}{8} \frac{\pi^2}{30} T_{\nu_e}^4 ,
\]

where \(T_{\nu_e}\) is the temperature of each neutrino species. Note, that since each neutrino species is slightly heated by the \(e^+e^-\) annihilation before it decouples at a different temperature, \(\Delta N_\nu = 0.046\) even in the standard big bang.

The dark radiation arising from the RS model is different from the other possible “dark” relativistic particles (e.g. sterile neutrinos). Indeed, during the BBN epoch the dark radiation in the RS model is nearly equivalent to an effective neutrino species. However it acts differently on the CMB. Whereas light neutrinos or non-interacting particles can stream and gravitate, a dark radiation term remains uniform everywhere. Thus, as clarified below, there is a cosmological sensitivity to either relativistic or light neutrinos at the CMB epoch, particularly given the fact that their number density is comparable to that of CMB photons. A dark radiation term of the form of interest here, however, has a different effect on the CMB.
This high density of free streaming particles can inhibit the growth of structure at late times, leading to changes in large scale structure (LSS) that can be constrained by the CMB and matter power spectrum. In particular, the number of neutrino species primarily affects the CMB by altering the photon diffusion (Silk damping) scale relative to the sound horizon. The sound horizon sets the location of the acoustic peaks while photon diffusion suppresses power at small angular scales. This affects both the Integrated Sachs-Wolfe (ISW) effect and the look back time. Hence, the effect of RS dark radiation is not equivalent to adding $\Delta N_\nu$ neutrino species. Moreover, if the added neutrino has a light mass, the CMB constrains that mass through its effect on structure growth in two ways: 1) the early ISW effect, and 2) gravitational lensing of the CMB by LSS. Indeed, a significant fraction of the power in the CMB on large angular scale is from the early ISW effect, but unaffected by the RS dark radiation of interest here. Hence, the CMB constraints on the dark radiation discussed here are not equivalent to the constraints on $\Delta N_\nu$ deduced in the Planck cosmological parameters paper.\textsuperscript{13} Although the Planck cosmological parameters paper\textsuperscript{13} mentions ‘dark radiation’, they only use that term in the sense of particle dark-radiation (similar to effective neutrinos) that can stream and gravitate, not as the effect of higher dimensional curvature.

To understand brane-world dark radiation consider the projected three-space Friedmann equation of a RS-II five dimensional universe:\textsuperscript{14}

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_N}{3} \rho - \frac{K}{a^2} + \frac{\Lambda_4}{3} + \frac{\kappa_5^2}{36} \rho^2 + \frac{\mu}{a^2} . \]  

(3)

Here $a(t)$ is the usual scale factor for the three-space at time $t$, while $\rho$ is the energy density of matter in the normal three space. $G_N$ is the four dimensional gravitational constant and is related to its five dimensional counterpart $\kappa_5$ by

\[ G_N = \kappa_5^4 \lambda / 48\pi , \]  

(4)

where $\lambda$ is the intrinsic tension on the brane and $\kappa_5^2 = M_5^{-3}$, with $M_5$ the five dimensional Planck mass. The $\Lambda_4$ in the third term on the right-hand side is the four dimensional cosmological constant and is related to its five dimensional counterpart by

\[ \Lambda_4 = \kappa_5^4 \lambda^2 / 12 + 3 \Lambda_5 / 4 . \]  

(5)

Note that for $\Lambda_4$ to be close to zero, $\Lambda_5$ should be negative. Hence the spacetime is AdS5.

In standard Friedmann cosmology only the first three terms arise. The fourth term is probably negligible during most of the radiation dominated epoch since $\rho^2$ decays as $a^{-8}$ in the early universe. However this term could be significant during the beginning of the epoch of inflation.\textsuperscript{15,17}

The last term is the dark radiation.\textsuperscript{13,19} It is called radiation since it scales as $a^{-4}$. It is a constant of integration that arises from the projected Weyl tensor describing the effect of graviton degrees of freedom on the dynamics of the brane.
One can think of it, therefore, as a projection of the curvature in higher dimensions. In principle it could be either positive or negative. Although it is dubbed dark radiation it is not related to relativistic particles. Since it does not gravitate, flow or scatter as would a light neutrino species, its effects on the cosmic microwave background (CMB) is different than that of normal radiation. Nevertheless, since it scales like radiation, its presence can alter the expansion rate during the radiation dominated epoch.

2. Effects of Dark Radiation

Both types of dark radiation can affect the light element abundances produced during BBN. They also affect the angular power spectrum of the CMB. Altering the expansion rate changes the temperature at which various nuclear reactions freeze out. This leads to deviations in the final BBN light element abundances. We define \( \rho_{DR} \equiv (3/8\pi G_N)\mu/\alpha^4 \) as the energy density of the dark radiation, and parametrize \( \rho_{DR}/\rho \) to be the ratio of the energy density of the dark radiation to the total energy density in relativistic particles at 10 MeV (before \( e^+ - e^- \) annihilation). The corresponding changes in the BBN abundances and the CMB power spectrum are then computed.

Observations of the CMB and the Hubble expansion rate \( H_0 \) suggests the possible existence of an additional density in the form of dark radiation\(^{20,21} \) The effect of the altered expansion rate on BBN was first discussed by Ref.\(^ {22} \) This effect was further studied by many authors\(^ {3,23,24} \) However, such exotic relativistic particles that do not interact with normal background particles are not the same as the brane-world dark radiation. The effects of these exotic particles have been studied by numerous authors\(^ {25-27} \).

3. BBN constraint

The primordial light element abundance adopted in this study are from Ref.\(^ {4} \). Hence,

\[
Y_p = 0.2449 \pm 0.0040 \quad (2\sigma) .
\] (6)

The 2\( \sigma \) Deuterium constraint is

\[
2.45 \times 10^{-5} \leq D/H \leq 2.61 \times 10^{-5} .
\] (7)

For \( ^7\text{Li} \) we adopt the 2\( \sigma \) constraint of

\[
1.00 \times 10^{-10} \leq ^7\text{Li}/H \leq 2.20 \times 10^{-10} .
\] (8)

We do not consider the \( ^3\text{He}/H \) constraint. Because the effect of either type of dark radiation on BBN is mainly to alter the expansion rate, they are both constrained equally be BBN. One difference, however, is that brane-world dark radiation can be of either positive or negative sign\(^ {28} \) The inclusion of positive dark radiation increases the cosmic expansion rate and causes the nuclear reactions to freeze out.
at a higher temperature. As a result, the neutron to proton ratio increases, since the \( n/p \) ratio is related to a simple Boltzmann factor at freezeout,

\[
n/p = \exp (-\Delta m/T) ,
\]

(9)

where \( \Delta m = 1.293 \text{ MeV} \) is the neutron-proton mass difference, and \( T \) is the photon temperature. The increased neutron mass fraction from a positive dark radiation term increases the \( D/H \) and \( Y_p \) abundances. In addition, the faster cosmic expansion results in the freezeout of the deuterium destruction via the reactions \( ^2\text{H}(d,n)^3\text{He} \) and \( ^2\text{H}(d,p)^3\text{H} \) at a higher temperature. This also leads to a larger deuterium abundance. The abundances of \( ^3\text{H} \) and \( ^3\text{He} \) are larger for a positive dark radiation. This is because these nuclides are mainly produced via the reactions \( ^2\text{H}(d,n)^3\text{He} \) and \( ^2\text{H}(d,p)^3\text{H} \), respectively, and the deuterium abundance is higher. When the dark radiation is negative the opposite effect occurs.

Figure 1 shows the calculated light element abundances, \( Y_p \), \( D/H \), \( ^3\text{He}/H \), and \( \text{Li}/H \) as a function of \( \eta \). The solid green line is the result for the standard BBN with no dark radiation. The dot dashed black line and the dashed blue line show the results of BBN in which the energy densities of the dark radiation are \( +0.2\% \) and \( -1.2\% \), respectively, of the total particle energy density. The two lines correspond to the cases of the upper and lower limits on \( \rho_{\text{DR}} \) derived from the constraints on light element abundances. The horizontal lines correspond to the observational upper and lower limits on primordial abundances.

For the case of positive dark radiation, we find an increase of the \( ^7\text{Li} \) abundance for \( \eta \lesssim 3 \times 10^{-10} \) and a decrease for \( \eta \gtrsim 3 \times 10^{-10} \). We note that the primordial \( ^7\text{Li} \) nuclei are produced as \( ^7\text{Li} \) in the low \( \eta \) region and \( ^7\text{Be} \) in the high \( \eta \) region during BBN. A positive dark radiation term leads to a slight excess of the \( ^7\text{Li} \) abundance because \( ^7\text{Li} \) is produced via the \( ^4\text{He}(t,\gamma)^7\text{Li} \) reaction and the abundance of \( ^3\text{H} \) is higher. There is also less time for the lithium destruction reaction \( ^7\text{Li}(p,\alpha)^4\text{He} \). On the other hand, a positive dark radiation decreases the \( ^7\text{Be} \) abundance. The slight increase in the \( ^3\text{He} \) abundance results in a somewhat increased production rate of \( ^7\text{Be} \) via the reaction \( ^4\text{He}(^3\text{He},\gamma)^7\text{Be} \). However, the significant increase of the neutron abundance leads to an enhanced destruction rate of \( ^7\text{Be} \) via the reaction \( ^7\text{Be}(n,p)^7\text{Li} \). As a result, the final \( ^7\text{Be} \) abundance decreases. In either case, dark radiation does not affect the primordial lithium abundance sufficiently to solve the lithium problem without violating the \( ^4\text{He} \) and deuterium constraints. Hence, we presume that the lithium problem is solved by another means and do not utilize the \( ^7\text{Li} \) abundance as a constraint on dark radiation.

We estimate the likelihood for \( \rho_{\text{DR}}/\rho \) assuming a gaussian prior on the observational limits of \( D/H \) and \( Y_p \). From this we obtain the 1-\( \sigma \) bound on \( \rho_{\text{DR}}/\rho \) of \( -3.10 \pm 4.49 \% \). This corresponds to an \( N_{\text{eff}} \) range of \( 2.81 \pm 0.28 \). This is comparable to the BBN+\( Y_p + D \) value of \( 2.85 \pm 0.28 \) deduced in Ref. 4 for particle dark radiation.
Fig. 1. Light element abundances, $Y_p$, D/H, $^3$He/H, and Li/H as a function of baryon to photon ratio $\eta$. The red horizontal lines correspond to the adopted observational upper and lower limits on primordial abundances. The solid green line is the result for the standard BBN with no dark radiation. The dot dashed black line and the dashed blue line show the results of BBN in which the energy densities in dark radiation are $+6.2\%$ and $-12.1\%$, respectively, of the total relativistic particle energy density (at 10 MeV). The two lines correspond to upper and lower limits on $\rho_{DR}$ derived from the light element abundances. The vertical solid blue lines show the CMB constraint on $\eta$ from *Planck*.

4. CMB constraints

Although the epoch of photon last scattering is in the matter dominated epoch, there remains an effect on the CMB power spectrum due to the still significant contribution from relativistic mass energy and the effect of the uniform dark radi-
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atation term on the expansion rate and acoustic oscillations of the cosmic fluid. On the other hand, the CMB power spectrum is very sensitive to a number of other parameters that have little or no effect on BBN. Thus, to obtain a total constraint on the dark radiation contribution to energy density, we have performed a simultaneous fit to the $TT$ power spectrum of temperature fluctuations in the CMB. To achieve this, we have fixed most of the cosmological parameters to their optimum values and only varied the dark radiation content and $\eta$ in the fit. Fits were made to the Planck data using the CAMB code. In the limit of no dark radiation we recover the Planck value of $\eta = (6.10 \pm 0.04) \times 10^{-10} \ (1\sigma)$. For $\eta$ fixed by the Planck analysis, the 2$\sigma$ constraint from the CMB alone would imply $-6.2 \% < \rho_{DR}/\rho \ (10 \text{ MeV}) < 12 \%$.

We note, however, that the deduced dark radiation content is sensitive to the adopted value of $H_0$. In the present work we utilize $H_0 = 66.93 \pm 0.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Planck+BAO+SN) from the Planck analysis. However, a larger value is preferred from local measurements of $H_0$, and a larger value of $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ was obtained when adding a prior on $H_0$. Adopting this larger value would shift the inferred dark radiation constraint toward larger positive values. We prefer the lower value of $H_0$ deduced by Planck because this discrepancy between the local value and the CMB value would in fact be explained by the presence of dark radiation at the CMB epoch.

It is important to appreciate that adding a dark radiation term is not equivalent to adding an effective number of neutrino species to the CMB analysis. This is illustrated in Figure 2. The upper and lower panels of Figure 2 show the effects on the CMB $TT$ power spectrum of adding dark radiation vs. an effective number of neutrino species, respectively. This figure plots the usual normalized amplitude $C_l$ of the multipole expansion for the TT power spectrum as a function of the moment $l$. As can be seen on the upper figure, a positive dark radiation has only a slight effect, while a negative dark radiation term (red line) slightly increases the amplitude of the acoustic peaks due to the diminished expansion rate. However, a relativistic neutrino-like species can stream and gravitate. Therefore, it has the opposite effect of increasing the amplitude of the first acoustic peak for a positive contribution while decreasing the amplitude for a negative contribution. In addition, a relativistic species also shifts the location of the higher harmonics. Thus, it is important to re-examine the CMB constraints on the brane-world dark radiation term independently of any previously derived constraints on the effective number of neutrino species. In particular, the impact of brane-world dark radiation on the CMB is less than that of a streaming relativistic particle species. Therefore, the constraint from the CMB is less stringent. This is apparent in Figures 2.

Figure 3 shows the combined constraints on $\eta$ vs. dark radiation based upon our fits to both BBN and the CMB power spectrum. The contour lines on Figure 3 show the CMB 1, 2, and 3$\sigma$ confidence limits in the $\eta$ vs. dark radiation plane. The shaded regions show the BBN $Y_p$ and D/H constraints as labeled.

The best fit concordance shown in Figure 3 is consistent with no dark radiation
although in the BBN analysis there is a slight preference for negative dark radiation. A similar result was found in the previous analysis of Ref.\textsuperscript{25} However, the magnitude of any dark radiation is much more constrained in the present analysis. This can be traced to both the CMB and new light-element abundances. Also, it is worth mentioning that the value of $\eta$ deduced from the WMAP data is $(6.19\pm0.14)\times10^{-10}$, while the (WMAP+ BAO + $H_0$) data is $(6.079\pm0.09)\times10^{-10}$\textsuperscript{28} Hence, the BBN dark radiation constraint based upon the WMAP results would be nearly identical and would also have a slight preference for a negative dark radiation.
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5. Conclusion

In conclusion we deduce that based upon our adopted 2σ (95% C. L.) BBN constraints, brane-world dark radiation is allowed in the range of −12.1% to +6.2% ($\Delta N_\nu = -0.19 \pm 0.56$) compared to the range deduced in Ref. [28] of −123% to 10.5% based upon constraints available at the time of that paper. After taking into account the 2σ limits on the dark radiation from the fit to the CMB power spectrum, this region shrinks to a range of −6.0% to +6.2% ($\Delta N_\nu = -0.19^{+0.56}_{-0.18}$). However, if the higher helium abundance of Ref. [32] were adopted, the 1σ BBN constraint increases to $(2.63 \leq N_\nu \leq 3.38, \Delta N_\nu = -0.19 \pm 0.28)$. This is comparable to the values deduced by Ref. [13]. For $\eta$ fixed by the Planck analysis, the constraint on positive dark radiation comes from the upper bound on the $^4\text{He}$ mass fraction and the upper bounds on the D/H. The limit on negative dark radiation arises from the constraint on cosmic expansion rate at the epoch of last scattering (the CMB) and the lower bound of D/H.

We caution, however, that a larger value for the Hubble parameter could shift the allowed CMB range to a higher positive contribution of dark radiation. Similarly, a larger primordial helium abundance could also shift the BBN range to a higher positive contribution of dark radiation. For example, if the higher helium abundance of Ref. [32] were adopted, the allowed range increases to $+3.2\% < \rho_{\text{DR}}/\rho (10\text{ MeV}) <
12%. In this case the lower bound is from BBN and the upper bound is from the CMB.

Similarly, Figure 4 shows the sensitivity of the CMB deduced constraint on particle dark radiation to the value of the Hubble parameter. The contours of allowed dark radiation content are represented by contours centered on different values for $H_0$ based upon different priors in the Planck analysis, i.e. the value adopted in the present work: $H_0 = 66.93 \pm 0.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Planck+BAO+SN); or $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Planck+BAO+SN+$H_0$).

**Fig. 4.** Constraints on particle dark radiation in the $H_0$ vs. $N_{\text{eff}}$ plane. Contour lines show the 1, 2, and 3σ confidence limits for based upon our fits to the CMB power spectrum. Red lines are for $H_0 = 66.93 \pm 0.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and blue lines are for $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ based upon different priors in the Planck analysis as labeled.

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